AUDIOFRACTIONS: A SOFTWARE TOOL FOR GENERAL TEACHERS IN THE TEACHING AND LEARNING RATIONAL NUMBERS BASED ON MUSICAL METAPHORS

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Abstract

The present work on progress shows the design, implementation and assessment of AudioFractions, part of an educational software package called Sound Interactive Music-Mathematics conceived to aid mathematical learning through sound and music in Chilean Primary Education. Adopting a cognitivist theory based on social and situated learning of mathematics, we adopted a three-stage plan: 1) generating significant metaphors relating graphic representations of fractions to sound and music. 2) Implementing functional software prototypes and the didactic guidelines for teachers, which required an in-the-field successful usability evaluation. 3) Assessing the software. Pupils in risk of social exclusion at three priority public schools in Santiago de Chile (N=137; 9-11 years old) experimented with AudioFractions in a 60-minutes session. After, pupils filled a questionnaire with their perceptions about the software, included their emotions. Results of the four assessment dimensions show high scores and an excellent reception of AudioFractions. Evaluation indicates an excellent positive emotional balance as well: pupils felt more positive emotions than negative ones while working with AudioFractions. This emotional assessment could be an indicator of validation with respect to the assessment dimensions.

Keywords: Mathematical teaching-learning, music and sound metaphor, educational software, fractions and rational numbers.

1 INTRODUCTION

The use of resources for Math class is a recurring topic in research on mathematics education. The level of abstraction required for such contents lead teachers to seek strategies that help create examples of a certain concept, observe the application of a theorem or perhaps visualize a demonstration or procedure.

The very abstract nature of mathematical objects compels to teachers and researchers in the domain of mathematical teaching to search for strategies to facilitate transition from concrete to abstract domains, or to use concrete materials for making certain operations; may be, to visualize a demonstration or procedure through software. In that kind of research, it is taking in account a selection of problems and extra-mathematical concepts that are in contact to students' real life, as well as concepts belonging to other sciences or artistic domains.

This kind of investigation also considers selecting contexts which are extra-mathematical, of situations inspired in daily life, which apply to other sciences as well as art. In the same manner, metaphors are another widely used teaching resource. That is, the description of a mathematical concept by means of another, completely different element, not necessarily mathematical, that reflects its essence and is likely rooted in student's previous knowledge. In this search for resources and contexts, it is interesting to consider the advantages that music and sound provide, given their physical-mathematical components, which can constitute a source of mathematical teaching and learning situations for all levels of formal education.

In this paper we present partial results for an ongoing investigation inspired precisely on this relation between music and mathematics. This research consists of the design, implementation and evaluation of Sound Interactive Mathematics (MMSI, Musi-MatemáticasSonorasInteractivas), made up of a four software applications and its corresponding teaching/learning activities. This resource is designed in a way that students experiment and interact with sounds and musical sequences that keep a strong relation to certain mathematical concepts and skills.

One of these modules, namely AudioFractions, will be shown in greater detail. In this one, positive fractions of value less than one are associated to musical sounds, generated in relation to an initial fundamental tone. In the next pages, it will be put forth the theoretical framework that support this investigation; also, the module itself will be described and the result of its application to students from four public schools from Chile's Metropolitan Region. The data collected shows a positive response from students, leading us to conclude that greater attention should be given to research on the use of sound and music as a means to mathematical instruction.

2 METHODOLOGY

2.1 Design and implementation of AudioFractions

In order to implement software that aids mathematical teaching and learning in Chilean Primary Education, a framework was designed that allows the use of sound and music as working metaphors for mathematical concepts. A study of the most difficult math content for the pupils at Primary Education was carried out in order to select the ones to include in the software [1]. The results enabled us to prioritize the mathematical concepts, such as representation, interpretation and operation with rational numbers, which are the subject of this study. This aforementioned framework is resumed next.

The design stage was carried out with help of experts on both math and music education. Several brainstorming sessions were carried out using an interdisciplinary approach, specifically seeking metaphors that would relate sound or music to mathematics, centered on the latter. Once filtered and classified, these ideas made up working models (nonfunctional prototypes). Each model includedone sound-musical metaphor that supported a mathematical concept.

After elaborating the design guide for the models and implementing the software by means of iterative prototyping, four of the models were developed on Pure Data [2], one of them being AudioFractions (AF). The main goal of AF was to facilitate the representation, interpretation and operation with fractions associating a graphic representation (bars) tosound pitch These representations were created from the idea of the pan flute: several tubes of different length, each one producing a tone of different pitch and with blue color of the bars representing the length "Fig. 1". Pupils can modify the numerator by changing the blue part of the bar, so changing the length of the represented tube. In this manner, the selected fraction of the tube creates a sound whose pitch is the multiplicative inverse of the pitch corresponding to the tube's whole length.

2.2 Representation in AudioFractions

In the way previously described, AF proposes the interactive study of positive fractions that are less than one by means of an audiovisual representation, thus facilitating the association between representation -graphic bars-, musical sounds -conceptual support- and the related mathematical concepts -fractions, operations, equivalences-. The base element of the resource is the vertical bar representing the integer, which can be divided inup to 16 equal parts (denominator) in order to select before a certain number of those parts (numerator). AF introduces in a non-explicit way the notion of fraction as an expression that represents *the part-of-a-whole.*

From a musical perspective, the integer is represented by one blue bar that corresponds to the root sound that one air column can generate. Higher pitches are produced when pupils reduce the numerator, that is, clicking a smaller amount of the bar's blue segments. Graphically, the construction of a musical sound is given first by the number of parts in which the denominator divides the bar, and second by the number of selected parts expressed by the numerator. AF has four modes of music-mathematical activities, which are based in the division of blue bars (root sound) in equal parts "Fig. 1").

The activities of *Simplification Mode* have three levels of difficulty. Students must find out one, two or three sound fractions equivalent to a given one by means of simplification procedures (division).

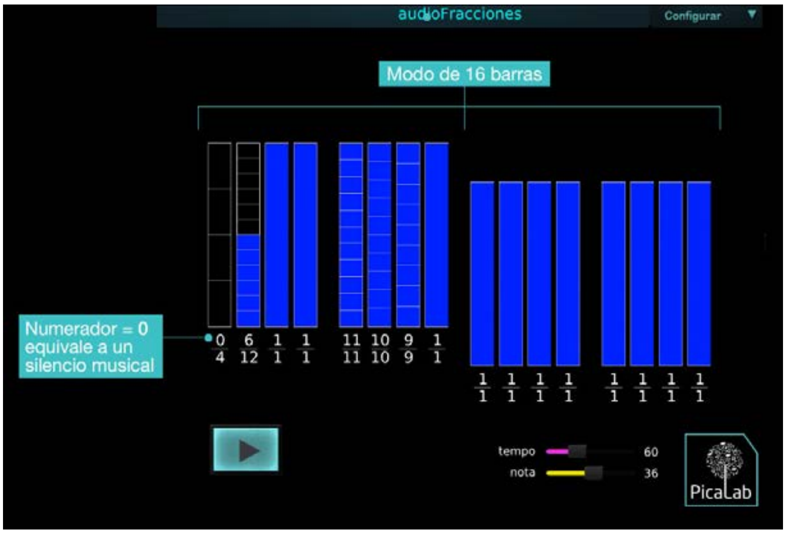

Figure 1. Sixteen-bars mode. This mode is similar to eight-bars mode. In it, students can freely manipulate a set of sixteen bars in order to generate melodic sequences (with or without rhythm). Numerator zero is equal to a bar-length silence.

2.3 Evaluation

The Third stage consisted in the evaluation of AF by both teachers and students of Priority Primary Schools (schools with children in risk of social exclusion) in Santiago's Metropolitan Area (Chile). Students' evaluation is shown next.

2.3.1 Pilot study

Previous to the software evaluation, a pilot study was carried out in a priority school of Santiago (n=40; 8-13 years old). This pilot was used to refine the assessment protocol as well as the data gathering instrument.

2.3.2 Sample

The sample for the evaluation stage was selected by means of probabilistic sampling among children of Priority Primary Schools of Peñalolén (Santiago's Metropolitan Area) (n= 137 aged 8-11). Three schools were selected "Table 1".

Table 1. Sampleofthis study, characterize by school,course and gender.

Sample per school	**N**
School A	25
School B	25
School C	87

Sample per course	**N**
3th. course	30
4th. course	63
5th. course	44

Sample per gender	**N**
Female	72
Male	65

2.3.3 Control of variables

In order to determine and verify the influence of potential variables on the pupils' evaluation, several items were included in the data gathering instrument (see below):

1) gender, 2) previous musical experiences (participation in non-school music activities), 3) previous mathematical knowledge (participation in extra-academic mathematical activities), 4) self-perception of mathematical competency, 5) self-perception of musical competency, 6) mathematical achievement (previous year's qualification); 7) music achievement (previous year's qualification), 8) self-perception of abilities related to the use of computers, 9) frequency of use of computers, 10) kind of use of the computer.

2.3.4 Data gathering instrument

A questionnaire was designed in order to collect data, which was validated by experts on the fields of mathematical education and music education (Alpha Cronbach= 0.952).The questions were expressed as affirmative sentences which were to be responded using a five-points scale. Each one of the five-points measurement scale was represented by one icon (a face), enabling pupils to express their opinions about AF by establishing a level of agreement to the sentence.

In the first part of the questionnaire, data on potential intervenient variables (see below) was collected in order to analyze its influence as covariable of the assessment results. The questionnaire included several assessment dimensions: 1) mathematical learning (8 items); 2) musical learning (4 items); 3) technical evaluation (5 items); and 4) global assessment (4 items).

Apart from these four dimensions, an affective-emotional dimension was included, that is, a control list with seven positive emotions and seven negative ones (unpaired). Students had to mark the ones that represented the way they felt while working with AF. Students who did not relate to any emotion from the list, should not mark anything. In data analysis, this affective-emotional dimension was correlated with the remaining assessment dimensions of AF. Finally, three open items were included in order to know what mathematical elements were learnt by pupils, the elements they liked the most and the elements they did not like.

2.3.5 Materials

For the evaluation, the following resources were used:

-PC computers with Windows running several hardware configurations in the computer lab of the schools were the evaluation was carried out.

- AudioFractions software

-Questionnaire

2.3.6 Procedure

The assessment was carried out in a sixty-minute session. Each student was working with one computer. The controllers (activity monitors) provided fast and clear instructions about mathematical contents and use of AF. After this, students put on earphones and began the first activity, which was guided by a question along the line of the Theory of Didactic Situations [3]. Once finished with this activity, they continued with the next one and so on. In the last ten minutes of the session, the assessment questionnaire was delivered. The controllers solved any problem or questions students might have while working with AF (mathematics, use of software, music, and questionnaire). All these issues were reported to the researchers in an ad hoc file.

3 RESULTS

Evaluation data was processed using IBM SPSS software, with a confidence level of 95%. No significant correlations were found between the aforementioned potential variables of influence and the dimensions of evaluation. Following, the results on each of these dimensions are shown

3.1 Global assessment

Global assessment dimension was determined by 4 items of the questionnaire. Statistical analysis shows an average score of 4.30, with biased positive scores and a discrete dispersion "Fig. 2"

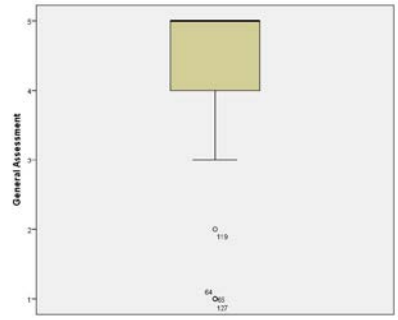

Figure2. *Globalassessment of Audiofractions*.

3.2 Assessment of mathematical learning

Pupils' perception about their mathematical learning was determined by 8 items of the questionnaire. Statistical analysis shows a high average score (4.35) with biased positive scores and low dispersion of values "Fig. 3"

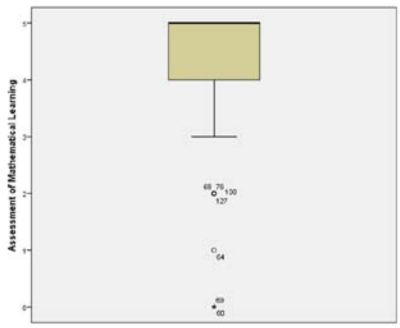

Figure 3.Students'scores on perceptions about Mathematical Learning with AudioFractions.

3.3 Assessment of music learning

Music learning dimension was determined by 4 items of the questionnaire. Data show a high average score (4.49) and low dispersion of values "Fig. 4"

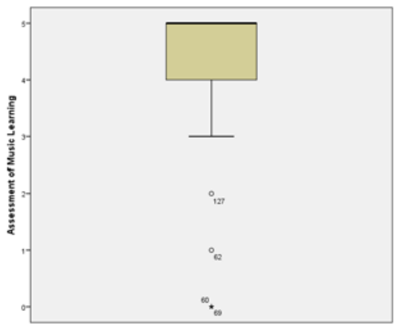

Figure 4. *Students' scores on perceptions about music learning with AudioFractions*.

3.4 Technical assessment

Technical assessment dimension was determined by 5 items of the questionnaire. Data shows a high average score (4.29), with biased positive scores and low dispersion of values “Fig. 5”

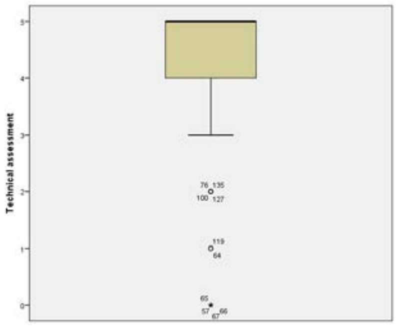

Figure 5. Technical assessment of AudioFractions

3.5 Assessment of the affective-emotional domain

As previously mentioned, emotional assessment was based on a control list of 7 positive emotions and 7 negative emotions (unpaired, no semantic scale was used). Results show pupils felt more positive emotions than negative ones, which indicate a good evaluation of the affective-emotional domain “Fig.6”, even though 15% of pupils did not answer this item.

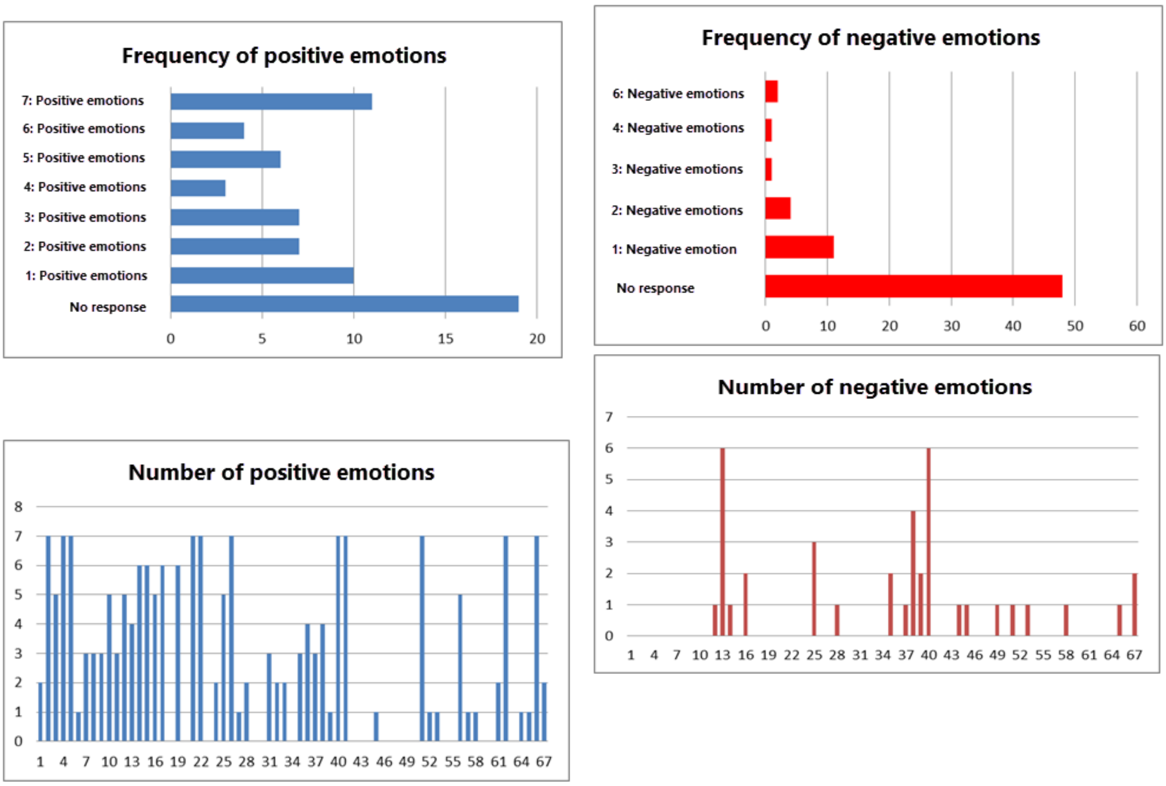

Figure6. Frequency and number of emotions perceived by pupils in the assessment of AudioFractions. Vertical axis represents the number of emotions felt by students. Horizontal axis representsthenumber of cases.

According to the data obtained, some statistically significant direct correlations at medium and low degree of association can be observed “Table 2”.

Table 2. Correlations between positive emotions and the dimensions of assessment of AudioFractions.

	Rho Spearman	P value
Correlation positive emotions and global assessment	0,261	**0,002**
Correlation positive emotions and mathematical learning	0,174	**0,042**
Correlation positive emotions and music learning	0,180	**0,035**
Correlation positive emotions and technical assessment	0,193	**0,024**

Also, data shows some statistically significant inverse correlations at medium and low level of association “Table 3”

Table 3. Correlations between negative emotions and the dimensions of assessment of AudioFractions.

	Rho Spearman	P value
Correlation negative emotions and global assessment	-0,167	0,051
Correlation negative emotions and mathematical learning	-0,199	**0,020**
Correlation negative emotions and music learning	-0,247	**0,004**
Correlation negative emotions and technical assessment	-0,097	**0,258**

Finally, reports from controllers gave extra perception about students’ attitudes towards AF. Basically, all of them were positive. Students showed motivation to: 1) use the program; 2) solve the controllers’ questions. These reports describe a distended context in which students asked for questions related with mathematics and sound and music, and particularly questions which showed greater insight into both the musical and mathematical concepts than those that arise in “normal” math classes. Questions regarding use of the program were barely asked to controllers. It follows that the interface was robust, functional and simple enough to be used by this kind of students.

4 CONCLUSIONS

It may be concluded that students from third and fifth grade of Primary Education in Chile considered it a positive experience to learn rational numbers through the use of sound and music by using AF software. The data collected shows an excellent disposition towards the program: all evaluated dimensions – mathematical learning, musical learning, global, technical, affective/emotional– show high scores.

The high scores on the cognitive and affective/emotional dimensions constitute, *de facto,* a triangulation of data: it is logical to think that there is a correlation between the value given by the student to a didactic mediator for learning of a certain subject –in this case, the AudioFraction software –and the positive emotions felt during its use. The program has provided a varied emotional experience, comprised to a learning process. The necessary actions for learning pick up the incidence of negative emotions typical to a process of effort and uncertainty when faced to a process of learning with a new tool. Therefore, the presence of negative emotions such as stress, tiredness, worry or anger is normal and expected. Nevertheless, positive emotions are much more widely present.

These results allow us to conclude that AudioFractions generates a greater level of emotional satisfaction than unsatisfaction, which can be observed both in the distribution of the areas of these variables and in the correlation with other dimensions of the evaluation. This produces a very positive vision of the emotional and regulatory processes developed. The incorporation of an affective/emotional analysis as criteria for evaluation seems pertinent and complementary to other criteria in the production of multimedia material with educational purposes both in formal and non-formal domains of education. This software gives teachers an important tool to tackle difficult concepts, such as fractions, with a better disposition by students, giving them greater chances of success in the teaching/learning process. Future study should be carried out to measure effective learning impact of the use of such a tool.

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